The Biomechanical Law of Linkage of Anterior and Posterior Guidance

Stomatognathic Law of Linkage

Dietmar Kubein-Meesenburg*
Hans Nägerl**
Göttingen, West Germany

IN SEARCH OF A BIOMECHANICAL LAW

A biomechanical linkage of the anterior with posterior guidance of the human stomatognathic system has been postulated by various authors (for detailed bibliography see 7, 9) despite the lack of the concrete law. The problem of this linkage has been approached differently and the efforts can be classified according to four types:

- a) empirical — statistical attempts
- b) qualitative formulations
- c) geometrical — cinematic computations
- d) qualitative formulations with possible conceptions.

Each type will be illustrated by one example.

a) In 1987 Kohno & Nakano5 approximate the anterior and the posterior guidance using chords to describe the position of the incisal edge related to the hinge axis in IP (intercuspidal position) and in EP (edge-to-edge positions). These points were connected to both guidances with a straight line and their inclination towards the occlusal plane measured (Fig. 1).

Based on the results in each case, the angle of rotation of hinge-axis-incisal edge was calculated and the coefficient of correlation of the entire random sample between anterior and posterior inclination was determined with \( r = 0.32 \). This low statistical relation indicated that the procedure of Kohno & Nakano is unsuitable for the characterization of the guidances. The inclination of chords is only mediately related to the shape of the guidance and to its curvature. Therefore, natural laws cannot be established in this manner.

* Prof. Dr., med. dent., Department of Orthodontics
** Dr. rer. nat., Institut of Physics
Fig. 1. Statistical examinations suggest a linear relation between anterior and posterior guidance but the positive relation within one human being cannot be determined in this manner.

Fig. 2. Slavicek approximated the protrusive posterior guidance by a polynomial of third degree.

Fig. 3a. Gysi² described the posterior and the anterior sagittal-vertical guiding contours as circular arcs around a stationary center of rotation R. The assumptions of the model are anatomically unsuitable.

b) Thielmann¹⁸ attempted in 1938 to transfer Hanau’s Quint⁴ into a “formula of articulation” that described the interdependence between the five articulation factors:
- inclination of the condyle path (K)
- inclination of the incisal path (I)
- inclination of the cups (C)
- inclination of the occlusal (Op) and
- curvature of the occlusal plane (Ok).
The formula

"equilibrium of articulation" = \frac{K \cdot 1}{O \cdot C \cdot O k}

is a simplification of the Hanau "Quint".

This "formula", in which the equilibrium of articulation is supposed to be a constant, has — as Thielemann himself explicitly emphasizes — no mathematical significance. It only puts together qualitative features. The line of reasoning concerning the dependences of the guidance does therefore not exceed qualitative argumentation.

c) Slavicek\textsuperscript{17}, in 1984, approximated the protrusive guidances by polynomials with four constants and formulated the relation by descriptive kinematics. This technologic description as it is used if the structural relation to a natural law is unknown and a physically unified theory is unavailable (Fig. 2).

d) In 1925 Max Müller\textsuperscript{16} concluded in his review of the hypothesis of Gysi\textsuperscript{1} (Fig. 3a), that assumed a constant center of rotation for the movement of the mandible: "The center (the axis) of the protrusive movement of our mandible is not stationary but moving". As long as 63 years ago Müller had thus formulated the correct concept that the movement of the mandible can be pictured as a rotation around a momentary axis of rotation which moves along the so-called "fixed holoide" through space during the movement of the mandibula (Fig. 3b). However, Müller was unable to establish a reliable form of this fixed holoide because the guidances were replaced by chords.
Fig. 3c. Kubein constructed for protrusion and for retraction the momentary rotational centers (rotational poles $R_{prot}/CO$, $R_{retr}/CO$) in CO. The arrows denote the direction of movement of the poles during protrusion resp. during retraction out of CO.

All four descriptive attempts have focused the relationship of anterior and posterior guidance by geometric kinematics. Therefore they remain within the descriptive phase without physical and biomechanical reasoning.

Kohno and Nakano have not used adequate quantities nor have they established a positive relation for both guidance within one patient. Their statistically determined values would represent, for therapeutic application, only the non-existing average mandible. Thielemann was vague despite the help of various parameters to coordinate the individual system. Slavíček has actually reproduced by a polynomial of third degree including four constants the shape of the posterior guidance with accuracy. He was able to reproduce the coordinates of any mandibular point for a certain condyle’s position if the corresponding rotation angle of the mandible was additionally measured. This is mathematic endeavor, some work for a computer, without substantial improvement because the computations neither reveal a structural feature of the cranial border movement nor provide a single criterion diagnosis.

Structurally considered, Müller was the most perceptive concerning the geometric description of the cranial border movements with the concept that the pure protrusive cranial border movement is a rotation around a momentary center moving along the “fixed holoide” during the movement of the mandibula. Müller has nevertheless used inadequate approximations for the geometry of guidance.

Kubein\textsuperscript{7} was credited with the first to accurately construct the
fixed holoide from the paths of guidance, beginning in centric occlusion (CO) for protrusion and retraction of the mandible (Fig. 3c). In addition, essential influences of growth and abrasion on the position of the fixed holoide in space have been discussed.

With the following considerations a second, biomechanical condition comes along which has been neglected by all four types of description: The cranial border function is only assumed at the presence of forces, even if weak, and only working when the masticatory muscles press the mandible against the maxilla. Kubein6,7,9 explicitly presented reasons for the forms of guidance influenced by the acting forces and also suggested the significance of the forces for the cranial border movement*. For example, engineering principles mandate routine description of the form of a bridge, and also relate the form to the forces exerted on the bridge.

Theorem 1: If the mandible in the cranial border is not accelerated, the reactions of support have to be perpendicular to the guiding surfaces. From this follows:

Theorem 2: The line of force from the resulting force vector, which is vectorially added up by the reactions of support, runs through the momentary center of rotation of the entire mandible. Otherwise expressed this means:

Theorem 3: The mandible in resting (e.g. in CO) only when the acting forces are equilibrium. This implies:

Theorem 4: That the vectorial sum of the forces and the vectorial sum of all angular momenta are zero.

The resulting angular momentum is equal to zero when the resulting force vector transgresses the momentary center of rotation of the mandibula because then it is without an effort arm. Its components are compensated in the supports by elastic counteracting forces.

States of equilibrium can be stable, labile or even indifferent. If the mandibula in the cranial border believes that in any position it may not assume a labile state it follows that the fixed holoide of the cranial border movement have a predetermined course and that a certain relation between the shape of the anterior and the posterior guidance exists. This relation has just been published9,11,12. A labile mechanical state would have the consequence that the masticatory muscles have to balance perma-

* A resolution of the total force in reactions of support is not yet a discussion of the significance of the forces for the cranial border movement.
Fig. 4. In protrusion only the mandibular incisal edge rotates starting ideally from the base point Bp around a stationary center A that is identified to the maxilla.

nently to sustain the mandibula e.g. in CO. Because of transit-time effects of the nervous feedback it would lead to permanent (control) oscillations of the mandible around its position of equilibrium.

The following will
(1) — explain the mechanical structure of the stomatognathic system in the cranial border
(2) — discuss questions of mechanical stability
(3) — demonstrate some features of the system and
(4) — present the law of linkage between anterior and posterior guidance describe its significance for incisal reconstruction.

THE MECHANICAL STRUCTURE

1. Shape of Anterior and Posterior Guidance
In pure protrusive movement a sagittal contour through the palatal concavity represents an anterior guiding curve of the mandible. This curve can be described by a catenary \( y = a \left( \cosh \frac{x}{a} - 1 \right) \) with a vertex that lies in the turning point, the base-point Bp. The sagittal, protrusive contour curves of humans are described by one and the same catenary\(^{10}\). Since catenaries in a relatively wide range can be approximated by their circle of curvature in the vertex, the sagittal anterior guidance starting ideally from the base-point is considered as a circular arc (Fig. 4). This circular arc around the stationary center A has the radius \( R_1 \). \( R_1 \) is equal to the radius of the circle of curvature “a” in the vertex of the catenary: \( R_1 = a \). In protrusive border guidance the mandibular incisal edge rotates around this stationary center of the maxilla A. As Fig.
Fig. 5. In protrusion the hinge axis C rotates starting from CO (CR in coincidence with CO) around a stationary rotational center B associated to the maxilla.

4 depicts, only the mandibular incisal edge rotates around A, but not the mandibular incisor.

During protrusive, cranial border movement in the TMJ the minimal thickness of the disk remains — because of biomechanical reasons — in constant relation to the guiding structure of the os temporale and of the eminentia including the condylus. Both guidances rotate in the same direction7. This is the reason that the distance between the sagittally cut contour of the eminentia and the corresponding hinge axis point remains nearly constant. The protrusive hinge axis trajectory can be referred to an enlarged image of the eminentia contour (Fig. 5)6,7. Like the structures of the os temporale the hinge axis trajectory can also be approximated by a catenary or more simple by its circle of curvature12. Corresponding circular movements of the hinge axis have been described by others13,14,15. The radius of the hinge axis trajectory is consequently composed of the radius of curvature of the eminentia starting from the CR-position, the minimal thickness of the disk and the corresponding radius of the condyle6. The sagittal hinge axis trajectory is not only the result of the guiding structures of os temporale (eminentia), disk and condyle, but also all sagittal guidances of TMJ.

2. The Biomechanical Linkage of Anterior and Posterior Guidance in Protrusive Cranial Border Guidance as Link Quadrangle

In pure protrusion, the rigid body "mandibula" carries out a plane movement. Such a movement is completely described if the track curves of two points of the mandibula are known. This is the case here:

— An incisal point moves along the sagittal contour curve of
Fig. 6. In protrusion the distance $L_{MD}$ between the incisal edge D and the hinge axis C remains constant. $L_{MD}$ rotates with every end (D resp. C) around stationary rotational centers (A resp. B). The distance $L_{MX}$ between A and B is stationary.

palatinal concavity (anterior guidance) and  
— the hinge axis point moves along the protrusive hinge axis trajectory (Fig. 6).

Physically considered both points are certain points of the mandible during sagittal movement. Their distance $L_{MD}$ certainly remains constant during cranial border movement:

$L_{MD}$ represents the functional length of the mandible.

A rigid body with two points respectively transversing a clearly determined path is identical to a *cam gear with positive drive* of kinematics. Cam gears turn into *link quadrangles* if the two guiding curves are considered as circles (Fig. 7). This is given with good approximation for the cranial border movement of an eugnathic stomatognathic system: The mandible rotates, starting protrusively from centric occlusion (CO), with two functional ends on a stationary circle of curvature. One end, the lower incisal edge, rotates anteriorly with the initial radius of curvature $R_1 = a$ of palatinal concavity. The hinge axis point moves posteriorly on a circular arc with the radius $R_2$.

This link quadrangle “maxilla-mandibula” of cranial border movement functions only when the mandible is muscually pressed on its cranial guiding paths. Technically, the stomatognathic system represents a *link quadrangle with closed linkage*. This closed linkage limits the movement of the gear within the function of cranial border guidance. The link quadrangle “stomatognathic system” is only responsible for a part of the movement of an equivalent link quadrangle and its joints are designed as technol-
Fig. 7. The movement of the mandible can be viewed as the movement of a couple in a link quadrangle.

Fig. 7 illustrates the mechanism of mandible movement. The diagram shows a link quadrangle with fixed support and movable couple. The movement of the mandible is represented as a couple rotating around a central hinge axis.

The diagram is labeled with various symbols:
- Link: $L_{MX}$
- Support (fixed): $L_{MD}$
- Couple (movable): $R_2$
- Points: A, B, R1, R2, Ao, Bo
- Angle: $\alpha$

According to nomenclature for gear engineering, such a link quadrangle (Fig. 7) consists of:
1. A fixed support
2. The two rotating links
3. The movable couple

These are connected by bearings. The links $R_1$ and $R_2$ rotate or oscillate around the endpoints A and B of the support. The free ends C and D guide the couple connecting these two points.

The length of support AB = $L_{MX}$ corresponds to the distance of the initial centers of curvature of palatal concavity and protrusive hinge axis. This length of support $L_{MX}$ is identified anatomically to the maxilla: so $L_{MX}$ is the functional length of the maxilla.

The links are depicted by the two radii of curvature $R_1$ and $R_2$. The length of the couple $L_{MD}$ is identical to the distance of incisal edge-hinge axis and corresponds to the functional length of the mandible.

**MECHANICAL STABILITY**

1. Equilibrium
   The closed linkage of the link quadrangle maxilla-mandibula implies that in its two bearings of posterior and anterior guidance only forces can be applied in a state of rest on the mandible perpendicular to the guiding curvatures. Otherwise, the mandible would be accelerated. Hence follows: The radii AD resp. BC of the rotating links lie on the lines of action of the two forces in the bearings (Fig. 8) that intersect in P, the momentary center of rotation of the mandible. The line of force of the resulting force F...
Fig. 8. The reactions of support $F_D$ and $F_C$ are perpendicular in D and C to the guiding contours. The lines of action of force run through the momentary pole P and add vectorially to the total force $\overrightarrow{F}$. $\overrightarrow{F}$ is generated by the masticatory muscles.

Fig. 9a. Throttle crank: The fixed holoide of a throttle crank can lead to a stable equilibrium: A random protrusive movement of the mandibula — a counter-clockwise rotation around P — leads to a clockwise angular momentum because of the moving pole. The mandibula is already stabilized by the resulting force vector, independent of sensitive control.

Fig. 9b. Double rocking shaft: The equilibrium is unstable in any case: Protrusion, counter-clockwise rotations around P lead to angular momenta which rotate in the same direction that accelerate the mandibula from CO in direction of protrusive dead position (physiologically impossible).
runs through P. The mandible is in a state of equilibrium. No angular momentum acts on it (in position of CO). Likewise, an equilibrium exists in any protrusive position.

2. Stability and Fixed Holoide
If the mandible moves anteriorly out of CO its momentary center of rotation moves along the so-called fixed holoide (Fig. 9a) which has been mentioned above. Therefore for each single protrusive position an other momentary center of rotation of the entire mandibula exists.

Two basically different types of link quadrangles with characteristic differences between the fixed holoides exist:

**Throttle cranks and double rocking shafts.**

In throttle cranks the smaller, anterior system of rotation around A can at least theoretically rotate around 360° while the posterior center B oscillates with the angle $\Delta \mu$. The corresponding fixed holoide can therefore only run through the posterior center B, whereas in double rocking shafts the fixed holoide can also run towards resp. away from A. Since under the condition of closed linkage the resulting force vector F has to lie in the sector of dial APB (see Fig. 8), stable equilibriums can only be achieved in throttle cranks: In the stomatognathic system random deviations from the state of equilibrium lead to retroactive, i.e. stabilizing angular momenta into CO (Fig. 9a). On the other hand, only labile states occur in double rocking shafts (Fig. 9b).

**HUMAN STOMATOGNATHIC SYSTEMS ARE GENERALLY THROTTLE CRANKS AS CAN BE PROVED BY INDIVIDUAL MEASUREMENTS!**

3. Shape of the Fixed Holoide of Cranial Border Movement
In cranial border function the real human “stomatognathic link quadrangles” only use a part of the range of movement of a technically equivalent link quadrangle, hence only a part of the theoretically possible 360°-rotation of the anterior rotational system. From the biomechanical ideal position (CO) (couple and anterior connecting rod AD are perpendicular to one another) the momentary rotational center moves cranially to infinity in order to come back caudally from infinity (Fig. 9a). Remarkable about the fixed holoide of the throttle crank “stomatognathic system” is that within the range of its function of real cases the difference from its asymptote is negligible. That means, within the actual range of function of the protrusive guidance the fixed holoide approximately corresponds to a straight line (Fig. 9a). Thus, within the complete range of function of cranial border guidance the resulting force vector only needs to change its direction slightly in order to cause an equilibrium in any position of the mandible.
The mandible therefore is positioned in a slightly stable, almost indifferent state. Small additional forces are able to move the mandible easily from place to place although the pressing force of the muscle depending on the momentary function can be very large.

SOME FEATURES OF THE SYSTEM

1. Constancy of Collineation

In ideal CO the point of collineation $H$, the intersection of couple $L_{MD}$ and support $L_{MX}$, is in its most distal position (Fig. 10). The theory (derived in 3) says that the relation of the distances $HD$ to couple $L_{MD}$ is almost independent of angle $\alpha$ between the initial inclination of the hinge axis trajectory and the couple. Therefore, the following formula

$$\frac{HD}{L_{MD}} = \frac{tg\varphi}{tg\alpha} = \text{const.}$$

is valid for human stomatognathic systems, whose angles are according to experience limited to the range of $0 \leq \alpha \leq 45^\circ$. Fig. 11 illustrates the relation $HD/L_{MD}$ depending on angle $\alpha$ for all stomatognathic systems of mammals which can be described by link quadrangles.

The ideal starting position in the anterior guidance has a special biomechanical significance regarding the collineation at the base-point of palatal concavity. The ideal positioning of the mandibular incisal edges in the "base-point" of palatal concavity is a special position even considering the collineation of the entire system. The point of collineation or the point of intersection of the support line and the line of couple, oscillates with each revolution of the gear. The ideal starting point is the most distal point of intersection that is possible in relation to the anterior guidance. This assignment conditions that during the initial phase of guidance the point of collineation virtually does not move. This special example illustrates that structurally in the assignment of single elements and in the linkage of the elements of guidance different biomechanic laws are combined so that the entire system works effectively.

2. Fixed Holoide and Constancy of Collineation

In Hain's text of gear engineering (3, p. 62) the angle between the straight line of connection of the point of collineation $H$ and of the momentary rotational center $P$ and the anterior polar line $AP$ ($\angle HPA = \varphi$, Fig. 10) is identical to the angle between the posterior polar line $BP$ and the tangent to the fixed holoide in the momentary pole ($P$). Thus, without the fixed holoide in detail, its course in the momentary rotational center is given, and with that it is possible to decide whether the concrete system is stable. It is stable if this tangent in $P$ is caudally intersected by the result-
Fig. 10. θ = HPD = φ = θ = BPC

Fig. 11. The constancy of collineation: The relation

\[ \frac{L_2}{L_{MD}} = \frac{HD}{L_{MD}} = \frac{\tan \phi}{\tan \alpha} \]

is in a wide range independent of angle \( \alpha \), the initial inclination of the hinge axis trajectory to the line of the couple.

The Biomechanical Law of Linkage of Anterior and Posterior Guidance

1. The Formula

From the requirements of equilibrium and from the dissimilar terms

\[ R_1/L_{MD} \ll 1 \quad (1) \]

The Journal of Gnathology Vol. 8, No. 1, 1989
Fig. 12. To the definition of the angle of oscillation $\Delta \mu$: $\Delta \mu$ is given by the anterior and posterior dead positions.

the following is derived\(^{11}\):

$$\frac{R_2}{R_1} = \frac{1}{\sin \Delta \mu/2} \cdot \frac{1}{\sqrt{1 - \sin^2 \alpha \cos^2 (\Delta \mu/2)}}$$  \hspace{1cm} (2)

$\Delta \mu$ is the angle of oscillation, the amplitude with which the posterior connecting-rod oscillates (Fig. 12). The collected data of patients infer that for the human stomatognathic system this angle of oscillation $\Delta \mu^{9,11}$ has the value:

$$\Delta \mu \approx 23^\circ$$  \hspace{1cm} (3)

The anterior-posterior mobility of the TMJ is measured by $\Delta \mu$, the angle of oscillation, as a general constant for man, comparable to blood-pressure or body-temperature. For man the equation (2) is therefore reduced to

$$r = r(\alpha) = R_2/R_1 = 5 \cdot \frac{1}{\sqrt{1 - 1.04 \sin^2 \alpha}} \approx \frac{5}{\cos \alpha}$$  \hspace{1cm} (4)

2. The Graph and the Determination of the Individual "a" from the Axiographic Path

Fig. 13 illustrates the graph of the function $r = r(\alpha)$ in polar coordinates. It is virtually a horizontal straight line. This presentation includes the possible linkages between the anterior and the posterior guidance, the possible range of $R_2$ and of the guidance protrusively out of CR (CR in correspondence with CO of the mandibula).

If reconstructive measures of maxillary incisors are necessary and the palatal constant "a" of the patient cannot or cannot exactly be determined from occlusion, then the "a" can be determined with axiographic measurements (Fig. 13).
Fig. 13. The presentation in polar coordinates of the relation of anterior and posterior guidance (details in the text).

Starting from the CO-point on the axiograph the radius of the circle of the axiographic protrusive curvature $R_2$ can be determined. In addition, the angle $\alpha$ that is the angle of inclination of the initial guidance to the plane of couple is measured, hence to the line mandibula - incisal edge - hinge axis ($L_{MD}$).

Example:
The point of intersection on the graph is determined with the side of the measured angle. Starting from this point of intersection, the radius vector in direction of $R_2$ is traced to the left and the value of $R_2$ is determined by interpolation. For example, the incisal "$a$" of 3 mm belongs to $\alpha = 34^\circ$ and a measured $R_2 = 18$ mm (Fig. 13).

The presentation in polar-coordinates clarifies that the law of linkage is a matter of a standard system with an incisal $a = 1$mm as the fundamental unit. With an incisal $a = 1$mm the radius of the hinge axis trajectory can vary from 5mm to 7mm and correspondingly the initial inclination from $0^\circ$ to $45^\circ$. With $a = 1$mm an $\alpha = 0$ belongs to $R_2 = 5$mm, an $\alpha = 45^\circ$ belongs to $R_2 = 7$mm (maximum value). A doubling of the incisal "$a$" results in a doubling of the initial range of the radius of the hinge axis trajectory from 10mm to 14mm, a tripling of the incisal "$a$" conditions also a tripling of the values of $R_2$ with the same variation of. The relation of $R_2$ to $R_1$ determines the factor of magnification of the standard gear of the stomatognathic system.

THIS RELATION OF ANTERIOR AND POSTERIOR GUIDANCE IN POLAR COORDINATES PRESENTED IN FIG. 13 MAKES IT POSSIBLE TO COMPUTE THE RELATION OF ANTERIOR AND POSTERIOR GUIDANCE IN DENTAL
PRACTICE (without direct understanding of the physical relations).

DISCUSSION

The anterior and posterior guidances are related by the law of linkage. Therefore e.g. the path of the hinge axis point in relation to a functional plane of the gear is measured to $L_{MD}$ as initial inclination to the plane of couple of the mandibula (Fig. 7). So far, all measurements of the initial guidance related to arbitrarily chosen planes of the skull such as the Frankfurt Horizontal or the hinge axis infra-orbital-plane is functionally insignificant. Nevertheless, the angle between the plane of couple of the mandible and the Frankfurt Horizontal seems to be very stable with a value of $30^\circ - 33^\circ$.

The law of the gear and its meaning cannot be understood from the geometric description of the guiding surfaces alone, as it is suggested by methods presented in chapter 1. Especially, it cannot be explained whether specific geometric relations are medically-physiologically acceptable. Only the consideration of the system of forces, its effect and its criteria of stability provide guidance for therapy. An increase of the mobility of oscillation means an anterior shifting of point K (the point of intersection of polar tangent with $L_{MD}$) and therefore a system of greater instability. The present considerations have only been made for protrusion. For laterotrusion the same holds true: The geometric shape of the guiding surfaces is discussed in relation to stability without major obstacles but mathematics is “only” slightly more complicated.

These insights into the mechanical structure of the stomatognathic system allow more precise therapy, (1) especially in cases of illness of the TMJ, (2) to judge conservative, prosthetics or orthodontics and (3) orthodontic-surgical related to “compatability with mechanics”.

PROSPECTS

The Stomatognathic System of Mammals

The value of the angle of oscillation $\Delta \mu = 23^\circ$ seems to be a constant of the human body. Other mammals — as we suppose — make use of the same system; only their angle of oscillation has another value according to their ingestion. This theory should also hold true if no explicit incisal guidance exists as for the elephant. Fig. 14 shows such a cranial skeleton. The incisors are missing. The TMJ obviously possesses the same structure as the one of the human being. In CO the condyle is assigned to the transition fossa/eminentia. The hinge axis rotates in protrusion around a stationary system, the center of curvature of the eminentia. Incisors do not exist. The molars are virtually merged into one “tooth”. The
occlusal planes of the upper and lower teeth can be approximated by circles if cut sagittally. Here, in occlusion the larger circle of the lower molar contour is contacted from within by the smaller circle of the upper. During a protrusive cranial border movement this point of contact slides backwards. Fig. 15 schematically shows the entire arrangement. The posteriorly directed movement of the point of contact K means an anterior rotation of the center D of the mandibular circle around the stationary center A of the maxillary circle. The movement of the center D of the mandibular molar is therefore directly comparable to the movement of the incisal edge of the human being.

The path of the point D represents in case of the elephant a virtual "incisal guidance". During this protrusive border movement the distance between this center D and the hinge axis C certainly remains constant and represents in analogy to the human being the functional length of the mandible $L_{MD}$. The connection line of the stationary rotational centers A and B is the functional length of the maxilla $L_{MX}$. Like in humans each of the functional ends of
the mandibula rotates around a stationary center: anteriorly around A with the radius \( R_1 = KD - KA \) and posteriorly around B with the radius \( R_2 \), which is composed of the radius of curvature of the eminentia, of the minimal thickness of the disk and of the radius of the condyle.

The principle of the stomatognathic system of the elephant can also be considered as a link quadrangle. The exploration of its special features still needs more exact measurements of its morphology. These examinations seem to imply that stomatognathic systems of mammals might in different variations obey to this biomechanical linkage.

**ABSTRACT**

In protrusive movement with combined incisal and condylar guidance, the mechanical structure of the stomatognathic system can be interpreted as a link quadrangle with closed linkage. The acting forces are in an equilibrium. The discussion of mechanical stability and hence the discussion of the shape of the fixed holoiode makes evident that the human stomatognathic system resembles a throttle crank. Its fixed holoiode can be approximated by a straight line within its functional range. With the aid of the point of collineation, the mechanical stability resp. instability can be assessed for diagnosis. The physical and mathematical interrelationship between anterior and posterior guidance can be used to calculate the curvature of the palatal concavity from axiographic measurements. Other stomatognathic systems (e.g. the elephant) are governed by the same principles.

**REFERENCES**


Dr. Dietmar Kubein-Meesenburg
Georg-August-Universität Göttingen
Klinik- und Poliklinik für Zahn-, Mund- und Kieferheilkunde, Abteilung Kieferorthopädie
Robert-Koch-Straße 40, D-3400 Göttingen, West Germany
Discussion: The Biomechanical Law of Linkage of Anterior and posterior Guidance, Authored by Kubein-Meesenburg, Nägerl

Hisao Takayama B.S., pH. D.  
Tokyo, Japan

In this paper, a stomatognathic law of linkage between anterior and posterior guidances is derived by means of gear engineering. Mandibular protrusive movement is simulated by link quadrangle, a kind of cam gear, providing that both initial guiding curves of anterior and posterior guidances are represented by circular arcs with radii of $R_1$ (1 ~ 6mm) and $R_2$ (5 ~ 42mm), respectively. The initial radius of curvature for anterior guidance, $R_1$, is equal to the parameter “a” in the formula of canaries, $y = a \cosh(x/a) -1$; derived experimentally by the authors for the contour curves of palatal concavities.

The authors derived the formula for the biomechanical law of the relation between anterior and posterior guidances as shown in equation (2) in this paper, which gives the ratio of $R_2$ vs. $R_1$ in terms of $\alpha$ and $\Delta \mu$. $\alpha$ is the angle between the initial inclination of protrusive condylar path and the line connecting the incisal point and center of the condyle (hinge axis in sagittal plane), and $\Delta \mu$ is the angle formed between both ends of the circular arc of protrusive condylar path and the center of the circle. Furthermore, the author says that $\Delta \mu$ is a general constant (~ 23°) for man based upon the data collected from patients. Equation (2) is reduced to a simple formula, equation (4). The authors believe that “a” (= $R_1$) for anterior guidance can be determined from the measured $\alpha$ and $R_2$ by using equation (4), when reconstructive measures of maxillary incisors are necessary and the palatal constant “a” of the patient cannot be determined exactly from occlusion.

The authors’ previous discovery of an empirical formula for the contour curves of palatal concavities is very valuable for the study of anterior guidance. However, as far as the descriptions in this paper are concerned, two points can be discussed as follows.

First, equation (2) may not valid since a different result was