The Ligaments of The Temporomandibular Joint

New insights into function, dysfunction and diagnostic possibilities of the T.M.J. through a combined mathematical and anatomical analysis

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INTRODUCTION

The most conspicuous disease of the T.M.J. is doubtless the internal derangement with its associated reciprocal clicking and opening limitation.¹

But this condition must be understood as a very advanced state of T.M.J. degeneration, whereas initial phases of the disease escape detection because of diagnostical difficulties.

The development of electronic devices for high resolution recording of the condylar paths has opened new diagnostic and therapeutic possibilities. We expect that initial degenerative states of the T.M.J. will reveal themselves as minor although detectable anomalies of the condylar paths and hope that these pathological conditions will respond quickly to adequate occlusal therapy, at least in those cases having a functional etiology.²,³,⁴

The diagnostic effort must concentrate in our opinion therefore away from the oseous and cartilaginous articulating surfaces to the other component of the joint, the ligaments, since both histological and clinical investigations⁵,⁶,⁷,⁸ demonstrate that the articulating surfaces do not show signs of noticeable damage except in advanced conditions.

By their very nature ligaments can only be susceptible either to shortening and elongation (including rupture) or loss of their elastic properties, the last condition affecting in the T.M.J. only the posterior fibers of the capsule, if any. Since the general task of ligaments is to serve as strengthening and movement-limiting structures of the joint, degenerative changes of the ligaments can affect only the limits of “free” condylar paths, and that is where we have to look for the corresponding path-anomalies. A new and promising field of diagnostic possibilities can be opened if we registrate “manipulated” condylar paths, for example the “transversal manipulated protrusion”, that is obtained by registrating the protrusion while transversally pushing the mandible to the right or
to the left.\(^4\) Here the condylar movement is restrained by the T.M.J. ligaments all along the path and degenerative changes of ligaments can give rise to conspicuous anomalies at any place along the recorded condylar path. With this technique we expect to observe many additional path-anomalies of diagnostic relevance.

But we confront the formidable problem of first identifying the exact restraining task nature has put on each one of the T.M.J. ligaments before we can engage in diagnostic evaluations.

Although there exist excellent anatomical descriptions of the T.M.J.\(^9\) the understanding of the role of all those structures the anatomist describes, especially the ligaments, is far from being complete.\(^10\) In our opinion the reason for this is that in contrast to an engineer's design, where stiffening elements, movement-restraining structures, and so on, will be placed at evident geometrical locations so as to minimize loads or to maximize the mechanical advantage ratios, nature must place structures subject to very strict spatial, developmental and histological constraints. Nature's designs are hence characterized by being usually poor mechanical devices with a complicated architecture where structures, notably ligaments, are being subjected to enormous loads resulting from very low mechanical advantage ratios, nature relying on healing processes to repair microscopic strain-damage of tissues that as the matter of fact will ensure even during function within physiological limits. Therefore nature's designs are not intuitive and not readily understandable for our technical minds, and we have to rely on detailed analysis to discover even their basic functional principles.

This paper deals with a detailed geometrical analysis that produced many interesting insights into the function and dysfunction of the T.M.J. Several new diagnostic approaches will be presented.

MATERIAL AND METHOD

An intuitive but realistic model of the T.M.J.\(^11\) can be described as follows: The articulating surface on the temporal bone functions as an impenetrable guiding-trough for the disc-condyle assembly. The ligaments serve as limiting and restraining elements of the condylar motion because of their given length and elasticity. We subjected this model to an accurate geometrical, computer-aided analysis.

Two eugnathic skulls with a complete natural dentition were investigated and biometrically evaluated. Several anatomical specimens of the T.M.J. were evaluated for comparison and reference.

Engaging the dentition in centric occlusion the missing disc was reconstructed with elastic impression compound. The hinge axis
was determined by a kinematic method and marked on the condyle. To describe the geometrical relations a system of coordinates was introduced as follows: The Y-axis was chosen to coincide with the hinge axis. The X-axis was chosen perpendicular to the Y-axis and for convenience running anterior-ventrally and parallel to the articular slope. The Z-axis was chosen to run perpendicular to both the X- and Y-axis in a posterior-caudal direction.

Anatomical questions were answered by our own experience, reference to anatomical specimens and consultation of anatomical literature.9,12,13

A sagittal cut through the eminentia exposed the sagittal section of the articulating surface, whose shape was approximated by a mathematical function.14 The shape of the transversal section of the fossa was taken into account in a phenomenological way by means of a mathematical relation reproducing the observed increase of the Z-coordinate of the hinge-axis mid-point during a transversal translation of the mandible.

The Y-coordinates of the relevant anatomical structures (origins of ligaments, etc.) were measured directly on the skulls, the X- and Z-coordinates indirectly on distortion-free photographs of the skulls.

The movement of the mandible was described mathematically as a superposition of a three-dimensional translation of the mid-point of the hinge-axis and two rotations of the mandible around the Y-axis (opening) and the Z-axis (mediotrusion).

The mathematical formulation of the model is to be found in the appendix. The first mathematical relations describe the movement of the mandible. The next relations are the formulation of the shape and impenetrability of the articulating surface on the temporal bone. Pathological bone configurations can be taken into account. The last relations describe the restraining of the mandibular movement by the limited length and elasticity of the ligaments. Elongation and shortening of ligaments can be taken into account.

This system of mathematical relations was solved numerically with a program written in Basic.

RESULTS

a) The Range of Motion. The lateral fibers of the capsule
Relying on clinical experience and careful evaluation of the abrasion facets in the dentition the range of protrusion, mediutrusion and mouth opening was estimated for both skulls.
Through preliminary calculations the factual length of each T.M.J. ligament was determined so that the calculated range of motion would agree with the estimated values. To start with we considered in our model the temporomandibular, sphenomandibular and stylomandibular ligaments and the posterior fibers of the capsule.

As a matter of fact, although the existence of the temporomandibular ligament has been many times described, one can not readily identify an isolated ligament when dissecting the T.M.J. and therefore one should rather speak of the temporomandibular ligament as consisting of fibers within the capsule of somewhat different histological structure than the capsular wall and that presumably developed for an outstanding functional task. Our calculations show, that if we divide the capsule into many elements, a region of the capsule, whose location is reasonably identical with the anatomist description of the temporomandibular ligament, actually turns out to play an outstanding role in the limitation of the retrusive, protrusive and transversal range of motion whereas two other capsular regions that we will denominate "most anterior lateral fibers" and "posterior lateral fibers" turned out to be essential to reproduce the expected range of opening and of mediotrusion, respectively (Fig. 1).

Moreover, no fully satisfying reproduction of the mediotrusive and opening range of motion could be achieved if these two additional capsular-ligaments were not taken into account.

The localisation of these ligaments is as follows:

The most anterior lateral fibers run from the lateral anterior angle of the condylar neck to well forward in to the lower and lateral angle of the processus zygomaticus of the temporal bone and thus into the next proximity of the posterior fibers of the masseter muscle. We will discuss this point later.
The posterior lateral fibers run from the lateral posterior angle of the condylar neck almost exact cranial to the lateral limit of the fossa mandibularis. The conspicuous upswing of the lateral fossa limitation in the vicinity of the insertion of these fibers is evidently a necessary osseous configuration to allocate posterior lateral fibers long enough to allow for sufficient range of motion of the mandible (Fig. 2).

b) The lateral manipulated protrusion. The most protruded opening
We have already spoken about the lateral manipulated protrusion and the relevance we expect its registration will have for the diagnosis of ligament alterations. Since the lateral deflection of the condyle is restrained by the T.M.J. ligaments, alterations of the length of the ligaments will directly repercute on the condylar path. Elongated ligaments will allow for an increased deflectivity of the condyle to lateral as compared with healthy ligaments. Our mathematical model allows to identify the ligaments restricting the lateral deflectivity of the condyle at any given point along the condylar path. This knowledge is the basis for the diagnostic evaluation of lateral manipulated protrusion paths.
The results are presented in a schematic drawing in fig. 3.

We see that the position of the condyles in centric occlusion and its close vicinity is defined by the temporomandibular ligaments of the right and left sides assisted by both "most anterior lateral capsular fibers". If the mandible protrudes somewhat under lateral manipulation, then only the temporomandibular ligament of the side where load is being applied will be engaged, whereas further protrusion will additionally bring the temporomandibular and the sphenomandibular ligaments of the other side into action. At the most protruded position of the mandible the stylomandibular ligaments joins in. Thus the lateral manipulated protrusion path of the mandible can be divided into four different path-segments each one being characterized by the ligaments involved, a circumstance that lends itself for diagnostical conclusions.

Calculations demonstrate that if the patient opens the mouth starting from the most protruded position, the temporomandibular, sphenomandibular and stylomandibular ligaments, that is, those ligaments that run caudal and posterior to the hinge axis will be relieved and thus allow for a further forward translation of the condyle along the articulating surface of the temporal bone past the eminentia, whereas the most anterior lateral fibers will obviously be tensed until they will finally limit mouth opening. The end position that the mandible attains is defined by the posterior fibers, the posterior lateral fibers, the temporomandibular and the sphenomandibular ligaments and last but not least the most anterior lateral fibers, the stylomandibular ligament being definitely not engaged in limiting the range of opening.

An extensive presentation of the diagnosis of ligament pathology based on the evaluation of high resolution recordings of the condylar paths will be given elsewhere.\textsuperscript{15} The general principle of the method is first to consider the range of motions, second to analyze the limits of the sagittal-caudal projection of the protrusive and opening paths, and then, most important, to look for anomalies in the lateral manipulated protrusion and other condylar paths, localizing with the help of our theoretical results the path-segment each anomaly can be assigned to and thus identify the ligament or ligaments that must be altered.

c) The lateral T.M.J. trauma
A very interesting clinical situation present patients that suffered a strong transversal blow against the mandible, an injury we will call lateral T.M.J. trauma and that is not uncommon with sportsmen. We restrict our attention to cases where bone fracture did not occur. The interesting point is that very often an unilateral irreversible reciprocal clicking will immediately appear, so that we expect to learn about the development of the internal derangement of the T.M.J. if we manage to understand what structures
The Ligaments of The Temporomandibular Joint

have being severed.

A strong transversal blow against the mandible causes a momentaneous transverse dislocation. Its consequences are susceptible to be analized with our model by simply calculating the elongation of the ligaments caused by a dislocation of a given amount. If we assume that elongation of ligaments up to 115% of their length will be reversible and more than that irreversible, whereas rupture will occur when ligaments will be strained to at least 150% of their normal length, then only the temporomandibular ligament of the side that was hit is susceptible to irreversible straining or rupture by lateral trauma, exception being made of those cases when the lateral dislocation surpasses 4 or 5 mm. But in those severe cases extended osseous damage must have originated, and they are outside the scope of our interest.

The result is easily understandable. By reference to fig. 3 we can identify the ligaments hindering lateral dislocation. The spheno-mandibular ligaments, the posterior lateral fibers and the most anterior lateral fibers can not be strained to an irreversible length, simply because they are very long compared to the amount of the dislocation. The stylo-mandibular ligaments are not engaged in restraining the dislocation of the mandible at all. And the temporomandibular ligament opposite to the side hit will not be subjected to excessive elongation. We postulate that the temporomandibular ligament will be severed at its common insertion with the disc into the condylar neck (Fig. 4), which explains why the disc will loosen and become displaced under the weak pull of the muscle pterygoideus lateralis superior to medial-anterior.

Thus we arrive at the result, that the irreversible elongation of the temporomandibular ligament or its rupture must be the only necessary and a sufficient condition for an immediate interior derangement of the T.M.J. and its associated reciprocal clicking to
develop.

This seems to be in contradiction to the usual idea the posterior disc attachment has to be severed for the so called “anterior dislocation” of the disc to develop. But let us consider another clinical situation known to be a predisposing condition for an internal derangement of the T.M.J., the class II division 1 dentition. It is a wide-held opinion, that with this condition the condyles are posterior dislocated. By reference to fig. 3 we see, that for this to happen, the temporomandibular ligaments must be elongated, whereas it is hard to imagine a mechanism which would have primarily severed the posterior disc attachment. Here again we arrive at the result, that the elongation of the temporomandibular ligament must be a sufficient condition for the internal derangement of the T.M.J. to develop.

d) The initial opening phases. The mediotrusion. The lateral manipulated mediotrusion.

We have already presented the ligaments involved in opening of the mouth starting at the most protruded position of the mandible and in limiting the maximal opening. Let us consider the initial opening phases. The first opening movement starting from centric occlusion can be a pure hinge axis movement if the patient tries to do so and most people are able to do it. This pure rotation of the mandible comes to an end when the temporomandibular ligament is completely tensed, and, as is well known, a passive forward gliding of the condyles along the articulating surface will follow.

An important consideration is to understand, that if the mandible of the patient is forced at its rearmost position, as is usual with Lauritzen’s hinge-axis-localisation method, the axis of rotation is no longer the geometrical center of the condyle. The mandible is forced to pivot around the insertion of the maximally tensed temporomandibular ligament in the condylar neck, or at least around a point in its close vicinity, so that the hinge-axis tends to be localized around 4 mm anterior and caudal of the geometrical center of the condyle with Lauritzen’s method.

When observing a mediotrusive movement it is often not clear to what extent the patient is protruding the mandible at the same time. It is therefore that we prefer to register and analyze “lateral manipulated mediotrusion paths”, that can be obtained by letting the patient start the mediotrusive movement of the mandible from the centric relation position while pushing the mandible transversally to the working side and thus suppressing protrusion. This condylar path is better suited for the diagnosis of ligament alterations than the “free” mediotrusive path, because the movement of the condyles is now continuously being restrained by the T.M.J. ligaments. The calculations deliver the very important result, that the transversal position of the mandible is being limited exclusive-
The Ligaments of The Temporomandibular Joint

Fig. 5. Lateral manipulated mediotrusive path of the right condyle of a patient with complaints of the left TMJ. The lower curve is the familiar sagittal-vertical projection of the condylar path. The upper curve is the Bennett-shift and is evidently abnormal.

Fig. 6. Lateral manipulated mediotrusive path of the left condyle of the same patient. The Bennett-shift is inconspicuous.

ly by the temporomandibular ligament of the working condyle at least during the first half of the mediotrusive movement. This means that mediotrusive paths deliver information about the working T.M.J. and not the balancing T.M.J. The immediate side shift, abnormal Bennett-movements, and so on, arise from ligament alterations of the T.M.J. of the working side, at least in those cases where an internal derangement of the balancing T.M.J. can be excluded.

To visualize these results we present in fig. 5 and 6 the lateral manipulated mediotrusion paths of the right and left condyles of a patient with complaints of the left and only the left T.M.J. The transversal movement of the mandible during mediotrusion of the left condyle is normal whereas the movement during the mediotrusion of the right condyle is abnormal, showing an enhanced deflectivity of the mandible to the working side due to elongation of the left temporomandibular ligament. A detailed discussion will be presented elsewhere.\textsuperscript{15}

The calculations of what is happening in the middle of the mediotrusive path are still inconclusive but they hint of a degeneration mechanism of the working T.M.J. The point is, that the sphenomandibular ligament of the working side seems to get tensed around the midpoint of the mediotrusive path, corresponding to a cuspid on cuspid position of the mandible, in those cases when the temporomandibular ligament of the working side has previously been elongated and admits for a transversal translation of the mandible to the same side. It is this lateral translation that puts the sphenomandibular ligament into action. Thus during latera excentric bruxism the mandible may be functioning like a
Fig. 7. Ventral view of a skull showing a hypothetical mechanism of TMJ degeneration. The mandible is in a half-way mediotrusive position to the right. If the right temporomandibular ligament has been previously elongated further straining through lateral eccentric bruxism may occur. T.m.: temporomandibular ligament. Sp.m.: sphenomandibular ligament. M.Pt.: lateral pterigoid muscle.

fulcrum-lever-device (Fig. 7), where the pivot-point is the lingula mandibularis of the working side, that is being held fixed by the sphenomandibular ligament. The force is being supplied by the anterior pulling muscles of the balancing side, and the effect is to tense the temporomandibular ligament of the working side. Since the sphenomandibular ligament is much stronger than the temporomandibular ligament and the mechanical advantage of the device is high, the straining potential will concentrate on the temporomandibular ligament and can be important. The tension on the sphenomandibular ligament has an important force-component to cranial and posterior, the direction this ligament is running, so that the reaction forces in the T.M.J. of the same side have an important component to cranial and posterior along the articular slope, that must be very deleterious for the joint. The clenching force of the muscles of the working side may come additionally to load the joint in the same undesirable direction.

The consequence of all this must be an elongation of the temporomandibular ligament and loosening of the disk attachment, very much the same way we have described before.

The medirotusive range of motion turns out to be limited by the posterior lateral fibers, as well as the sphenomandibular and the temporomandibular ligament of the balancing T.M.J.. Notice that these ligaments determine the end position of the mandible in the transversal direction, too. This implies, for example, that even when a Bennett-side-shift occurs due to an elongated temporomandibular ligament of the working T.M.J. the end point of the mediotrusion will be the normal one, as long as the balancing T.M.J. is healthy: its intact ligaments will pull back the mandible during the last phases of the mediotrusion thus compensating the Bennett-shift.¹⁵
A FURTHER DEVELOPED MODEL OF LIGAMENT FUNCTION

An observation that can be made on any patient is documented in fig. 8 and 9 that show, that during opening of the mouth or protrusion of the mandible the lateral pole of the condyle will project visibly through the skin. This phenomenon is of course not only visible, but can also be detected by palpation. Evidently, for this to happen, the lateral pole must project through the lateral fibers of the capsule and deform them. This deformation must have important consequences, since as we have seen, the temporomandibular ligament together with two other ligaments also localized in the lateral capsular wall play a decisive role in the T.M.J. function.

If we look closely at the joint from a posterior lateral position to judge the osseous configuration, we see that the temporomandibular ligament runs in a straight line when the mandible is in centric occlusion (Fig. 10). But in an intermediate opening or protrusive
Fig. 12. Schematic drawing describing a self-centering mechanism of the disc during opening and protrusion, based on the interaction between lateral pole and the temporomandibular ligament.

position however, the lateral pole projects to lateral and bends the temporomandibular ligament(Fig. 11). Since this ligament is composed of nonelastic fibers it is unable to stretch, so that the effect of this bending is comparable to shortening the ligament. The condyle will be pulled in the direction of the ligament against the articular surface of the temporal bone, squeezing the disc in between. This gives rise to a most important self-centering mechanism of the disc that we will consider in detail. If the disc happens to be slightly displaced then, because of its biconcave shape, its thinnest part will no longer be directly between condyle and articular slope. The bending of the temporomandibular ligament during opening or protrusion causes a squeezing of the disc, that will easily slip away and reposition at once, since the coefficient of friction in a healthy joint has a very low value, comparable to that of sheet ice(Fig. 12). In the mediotrusion this interaction between lateral pole and temporomandibular ligament does not arise, as can be verified by palpation of the lateral pole of, for example, the right condyle, because it keeps moving transversally to the left and away from the temporomandibular ligament until the restraining effect of this ligament together with the sphenomandibular ligament and the posterior lateral fibers is installed.

Let us try to understand nature’s intention when developing this device. To allow for a sufficient range of motion in mediotrusion the temporomandibular ligament must be longer than is needed to guarantee the protrusive range of motion. This has to do with the phenomenon we have just spoken about, that the condyle moves transversally away from the temporomandibular ligament in mediotrusion, but does not do so in protrusion. Thus were it not for this interaction between lateral pole and temporomandibular ligament, whose effect is analogous to shortening the ligament, the temporomandibular ligament would be too long at intermediate protrusive positions, and consequently the T.M.J. would demonstrate undesired distractivity.
THE DEVELOPMENT OF THE INTERNAL DERANGEMENT OF THE T.M.J. BY BRUXISM

Let us consider a situation where the self-centering mechanism of the disc is invalidated.

During bruxism and clenching there appear very high compression forces in the T.M.J. that originate histological changes of the surface of the disc. The important thing is, that the coefficient of friction of the altered disc will be much higher than normal, hence a lightly displaced disc would not be slippery enough to reposition when it gets squeezed during opening or protrusion. Consequently high tension forces in the temporomandibular ligament appear whose magnitude depends on the compressibility of the disc, the magnitude of the muscular forces and the amount of bending this ligament suffers, this is, the amount the lateral pole will project to lateral. We have calculated forces up to hundreds of kilopounds. Compressed under such high forces, even a rough disc can accidentally reposition with an audible popping sound, and that must explain the spontaneous, sporadic, isolated (this means, non-reciprocal) opening noise of the bruxist, a phenomenon that can not be explained by Farrar’s theory. But this high tension forces will slowly elongate the temporomandibular ligament presumably at its common insertion with the lateral attachment of the disc into the condylar neck (Fig. 4), causing a higher mobility of the disc. Under the weak pull of the muscle pterygoideus lateralis superior the disc will be further displaced to medial-anterior, jamming its thicker posterior rim further inbetween condyle and fossa, so that opening or protrusion will again originate high tension forces in the temporomandibular ligament. This process keeps on until the disc is totally displaced and reciprocal clicking is finally installed.

It is well-known that the lateral pole very often presents evidence for remodelling processes even in early stages of T.M.J.-degeneration. We consider this remodelling to be nature’s attempt to reduce the tension forces in the temporomandibular ligament by flattening the lateral pole.

HYPOTHESIS

Do the most anterior lateral fibers play an active role in stabilizing the joint during clenching? A hypothesis

We have already spoken about the remarkable fact that the most anterior lateral fibers run nearly parallel to the procesus zygomaticus and insert very much in front of the eminentia, in the close vicinity of the masseter. Some investigators have found that fibers of the masseter insert into the collagenous fibers of the joint capsule presumably in the region we denominate most anterior lateral fibers. Contraction of the masseter may pull at this fibers in a perpendicular direction. When the ligamentous fibers deflect in a caudal direction this results in a pull on the condylar neck to anterior (Fig. 13). This device is characterized by a very
Fig. 13. Schematic drawing describing a hypothetical mechanism of stabilization of the TMJ, based on the caudal deflection of the most anterior lateral fibers by some fibers of the masseter.

Fig. 14. Electronic recording of the position (+) the condyles attain during forceful biting on a flat anterior jig. The condyles have been previously manipulated in centric relation, which is at the origin of coordinates. Notice that the condyles slipped to anterior. The curves represent the protrusive (V) and the retrusive (R) paths of the right (left side of drawing) and left (right side of drawing) condyle. The upper curves are the sagittal-transversal projections, the lower curves the familiar sagittal-vertical projections. The retrusive paths (R) have been intentionally drawn off-place for clarity.

high amplification of the forces acting, so that a weak pull perpendicular to the direction of the ligament results in a high tension along the ligament. The net effect would be to assist forward acting muscles in stabilizing the joint against a posterior slipping of the condyle along the tilted articulating surface during forceful biting and clenching. The advantage of this device would lay in being to a wide degree insensible to neuromuscular discoordination and in the high mechanical advantage obtained.

Fig. 14 is a high-resolution recording of the position the condyles assume during forceful biting on a flat anterior jig. The position of the condyles is marked by crosses. The patient was first manipulated in centric relation and then asked to bite. When patients do so, the condyle will as a rule slip to anterior and not to posterior and cranial as one would expect. This phenomenon puts some evidence in the mechanism of joint stabilization through deflec-
tion of the most anterior lateral fibers since it seems impossible for patients with healthy joints to avoid slipping of the condyles to anterior.

**APPENDIX**

1. — Mathematical description of the movement of the mandible.
(Undashed variables: coordinates in centric occlusion)
(Dashed variables: transformed coordinates)

\[
\vec{M}' (i, r/l) = R + \Phi (\omega) \cdot \Psi (\psi) \cdot \vec{M} (i, r/l)
\]

where \( i = 1, \ldots, N \) denominates the ligament and \( r/l \) denote the right or left TMJ.

\[
\begin{pmatrix}
    x \\
    y \\
    z
\end{pmatrix}
\quad \rightarrow
\begin{pmatrix}
    x' \\
    y' \\
    z'
\end{pmatrix}
\]

\( \vec{M}, \vec{M}' \) denominate the coordinates of the insertion of the ligaments in the mandible.

\[
\vec{R} = \begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}
\]

is the translation-vector of the hinge-axis mid-point.

\[
\Phi (\omega) = \begin{pmatrix}
\cos(\omega) & 0 & -\sin(\omega) \\
0 & 1 & 0 \\
\sin(\omega) & 0 & \cos(\omega)
\end{pmatrix}: \text{Opening transformation.}
\]

\[
\Psi (\psi) = \begin{pmatrix}
\cos(\psi) & -\sin(\psi) & 0 \\
\sin(\psi) & \cos(\psi) & 0 \\
0 & 0 & 1
\end{pmatrix}: \text{Mediotrusion transformation.}
\]

2. — Formulation of the shape of the articulating surface and of its impenetrability for the disc-condyle assembly.

\[
z' \geq a \cdot y'^2 + A + B \cdot x' + C \cdot x'^2 + D \cdot x'^3 + E \cdot \exp (F \cdot x')
\]

The coefficients \( A, B, C, D, E \) and \( F \) are calculated by approximating the shape of the sagittal section of the articulating surface.

The coefficient \( a \) is choosen so as to correctly represent the
change of the z-coordinate of the center of the condyle when transversally translating the mandible by a given amount.

3. — Formulation of the constraining effect of the ligaments.

\[ \| \mathbf{M'} (i, r/l) - \mathbf{S} (i, r/l) \| \leq D (i, r/l) \cdot \| \mathbf{M} (i, r/l) - \mathbf{S} (i, r/l) \| \]

where \( i = 1, \ldots, N \) denotes the ligament and \( r/l \) denotes the right or left condyle.

\( \mathbf{S} (i, r/l) \) denotes the coordinates of the insertion of the ligament in the skull.

\( D(i, r/l) \) denotes the elongation of the corresponding ligament.

REFERENCES

The Ligaments of The Temporomandibular Joint

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